Interaction of Ultracold Antihydrogen with a Conducting Wall

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Abstract

We investigate the interaction of ultracold antihydrogen with a conducting surface. Our discussion focuses on the physical regime where the phenomenon of quantum reflection manifests. We calculate the reflection probability as function of incident atom energy. We find that, for ground state $\bar{H}$ atoms (with $T < 10^{-5}$ K), the probability of reflection is $R \simeq 1 - kb$, where k is the momentum of the atom and $b = 2174.0$ a.u. is a constant determined solely by the van der Waals-Casimir tail of the atom-wall interaction.

We show that quantum reflection, which suppresses the direct contact of ultra-cold atoms with the surface, allows for the possibility of confinement and storage of cold antihydrogen atoms. We calculate the life-time of confinement as a function of antihydrogen energy. We develop a theory of $\bar{H}$ in a wave-guide and propose its application to fundamental measurements. In particular, for measurement of retardation corrections in the long-range component of the antiatom - wall potential. We demonstrate, for $\bar{H}$ falling in the gravitational field of Earth onto a conducting surface, the existence of quantized $\bar{H}$ states. We calculate that the lifetime of ultracold $\bar{H}$ in its lowest gravitational state and obtain $\tau = (Mgb/2\hbar)^{-1} \simeq 0.1$ s, where Mg is a gravitational force acting on the antiatom. We propose that measurement of this lifetime may provide a new test for the gravitational properties of antimatter.

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I. INTRODUCTION

The phenomenon of quantum reflection in the ultra-cold atomic collisions has attracted lots of attention during the last decade. The propensity of an ultra cold atom, under the influence of an *attractive* atom-wall potential, to undergo total reflection has been predicted on quantum-mechanical grounds quite some time ago [1]. However, the observation of this counter-intuitive effect has only recently been observed in the laboratory [2], [3]. Because quantum reflection prevents atoms from reaching the wall, it suppresses inelastic atom-surface reactions and thus, it can be exploited as an important tool for manipulating cold atoms [4]. Another reason for interest in quantum reflection is that it occurs at large atom-wall distances where retardation effects are important. Retardation leads to the Casimir effect [5, 6, 7], where the van der Waals $1/z^3$ atom-surface potential melds into a $1/z^4$ power law potential at large distances. Quantum reflection is sensitive to the long-range component of the atom-surface potential [8] and may therefore allow new tests for the predictions of QED [3, 9, 10, 11].

Recent success in the production of the cold antihydrogen (further referred to as $\bar{H}$) [12, 13] has spurred renewed interest in this field. The ultimate goal of the antihydrogen project, initiated at CERN, is to enable accurate tests of CPT and gravitational properties of antimatter. Such a capability requires the availability of ultra-cold antiatoms. Quantum reflection may provide a new tool that hastens the realization of laboratory measurements. We will show that an interesting manifestation of quantum reflection, for antiatoms, is the existence of long-lived states of ultracold $\bar{H}$ in material cavities. We introduce a theory of wave-guides for anti-atoms and discuss their possible application in laboratory studies. Measurement of anti-atom life-times in a cavity may provide valuable information on the properties of the long range interaction between the antiatom and the surface. The measurement of the gravitational force acting on an antiatom poses as an intriguing possibility. An essential property of antiatom-surface interaction is annihilation of $\bar{H}$ on the surface. Anti-atoms that are not reflected are lost to annihilation and $\bar{H}$-wall annihilation events can be detected to measure the quantum reflection probability.

In Sec. II we discuss quantum reflection of $\bar{H}$ impinging on a perfectly conducting surface at normal incidence and present a calculation for the reflection probability. The latter is given as a function of incident energy, including the zero energy limit characterized by the

complex scattering length. In section III we study the probability for quantum reflection as function of the antiatom - wall distance. In Sec. IV we apply our theory and calculate the lifetimes of $\bar{H}$ enclosed in a material cavity. In Sec. V we discuss the passage of $\bar{H}$ through wave-guides. In Sec. VI we discuss the possibility for application of quantum reflection in measurements of the gravitational properties of an antiatom. Finally, in Sec. VII we study the loosely bound states of atoms confined in the atom-wall potential. We show that such states strongly affect quantum reflection when absorption by the wall is weak. We discuss the physical conditions corresponding to the limiting cases of weak and strong absorption (annihilation) on the surface and show that, in the latter case, the (anti)atom-wall states are destroyed.

II. QUANTUM REFLECTION AND ABSORPTION

Quantum reflection from an *attractive* (with a sufficiently large derivative) potential is known to manifest [14] at low energies. In our discussion we are mainly interested in those cases where the probability of quantum reflection is close to 1. The qualitative parameter associated with the quantum motion of the particle with mass m and energy E in the potential $V(z)$ is the local de Broglie wavelength $\lambda_B(z) = 2\pi\hbar/\sqrt{2m(E-V(z))}$. One expects quantum behavior when:

$$\frac{\partial \lambda_B(z)}{\partial z} \geq 1. \tag{1}$$

For a homogeneous potential, $-C_s/z^s$ with $s > 2$ in the limit of zero energies condition (1) is fulfilled for distances z such that

$$z_c \leq z \ll z_f \tag{2}$$

$$z_c \equiv (2\sqrt{2mC_s}/s)^{2/(s-2)} \tag{3}$$

$$z_f \equiv 1/\sqrt{2mE}. \tag{4}$$

Condition (2) defines the domain where the reflected wave is generated and is applicable if the energy of the incident atoms satisfies the condition

$$E \ll E_c = 1/(2mz_c^2). \tag{5}$$

A detailed discussion of the ultra-cold atom quantum reflection can be found in refs. [8, 15, 16]. The $\bar{\text{H}}$ – wall interaction is described by the potential, hereafter denoted $V(z)$. It is

not homogeneous and differs from that of the H - wall interaction. In particular it is purely attractive even at very short distances $z \leq z_s \simeq 1$ a.u. and is strongly absorptive at the origin due to the likely-hood of annihilation. However at large $\bar{\mathrm{H}}$ – wall separations $z \gg z_s$ it is dominated by induced dipole-dipole terms in the atom-wall interaction, similar to the case of the hydrogen - wall system. The potential for H impinging on a perfect conducting wall is known for large z [5] and will be hereafter denoted as $V_{CP}(z)$; in our treatment we use $V_{CP}(z)$ calculated in [17]. At distances $z_s \ll z \ll \lambda_\omega$ the potential has the van der Waals form

$$V_{CP}(z) \simeq -C_3/z^3 \tag{6}$$

with $C_3 = \frac{1}{4\pi}\int_0^\infty \alpha_d(i\omega)d\omega = 0.25$ a.u.^{2}. Here $\alpha_d(i\omega)$ is the dynamic dipole polarizability of the (anti)hydrogen atom, expressed as a function of imaginary frequency $i\omega$, and λ_ω is the effective wavelength that gives the main contribution to $\alpha_d(i\omega)$. For the distances $z \gg \lambda_\omega$ retardation effects are important and the potential is given by

$$V_{CP}(z) \simeq -C_4/z^4 \tag{7}$$

where $C_4 = \frac{3}{8\pi}\frac{\alpha_d(0)}{\alpha} = 73.62$ a.u.^{3}, $\alpha_d(0) = 9/2$ a.u.^{3} is the ground state static dipole polarizability of the antihydrogen, and $\alpha = 1/137.04$ is the fine-structure constant. We note that the above expressions for C_3 and C_4 are valid in the limit of perfect conducting surface [5], and can be specified for a realistic metal or dielectric surface at finite temperature [18]. At short distances $z \simeq z_s$ the interaction between $\bar{H}$ and the wall differs from that of H. In particular it includes an inelastic component, corresponding to the process of capture of $\bar{p}$ and $\bar{e}$ in the medium of the wall followed by subsequent annihilation. We take annihilation into account by imposing full absorption on the wall implemented in the manner described below.

The Schrödinger equation that governs the $\bar{H}$-wall scattering is

$$\left[-\frac{\partial^2}{2m\partial z^2} + V(z) - E\right]\Phi(z) = 0 \tag{8}$$

At distances $z_s \ll z \ll \lambda_\omega$ the potential has form (6) and the solution of Schrödinger equation is

$$\Phi(z) \sim \sqrt{z}\left(H_1^{(1)}(\rho) + \exp(2i\delta_s)H_1^{(2)}(\rho)\right) \tag{9}$$

$$\rho = 2\sqrt{2mC_3/z} \tag{10}$$

where $H_1^{(1)}(\rho)$ and $H_1^{(2)}(\rho)$ are the Hankel functions [19] of order 1, and $\delta_s = \delta_1 + i\delta_2$ is a complex phase-shift produced by the short range part of the interaction. The imaginary part of δ_s is due to annihilation of $\bar{H}$ on the surface. We used here the analytical form of the zero-energy solution in homogeneous potential $-C_3/z^3$ [20] and neglected the energy of the incident $\bar{H}$ since it is small compared to the potential $V_{CP}(z)$ at $z \simeq z_s$. In our treatment we are interested in low energies where the above requirements are always satisfied.

The condition of full absorption, which we apply, is

$$\delta_2 \gg 1. \tag{11}$$

This condition selects the "incoming wave" solution of the Schrödinger equation and suppresses the "reflected wave"

$$\Phi(z) \sim \sqrt{z} H_1^{(1)}(\rho) \ , \ z_s \ll z \ll \lambda_\omega. \tag{12}$$

One can check that in the limit of small z the above solution coincides with the semiclassical "incoming wave"

$$\sqrt{z} H_1^{(1)}(2\sqrt{2MC_3/z}) \to \frac{1}{\sqrt{p(z)}} \exp(-i \int p(x) dx) \tag{13}$$

where $p(z) = \sqrt{2mC_3/z^3}$ is the local classical momentum.

The full absorption condition $\delta_2 \gg 1$ leads to insensitivity of solution (12) in details of the short-range interaction since it is independent of the short-range phase shift δ_s. Thus, for strong annihilation on the surface the outgoing flux of $\bar{H}$ is determined solely by the asymptotic properties of the potential.

We impose (12) as a boundary condition in solution of the Schrödinger equation, which now does not depend on the short-range physics. The large z asymptotic form of such a solution is

$$\Phi(z \to \infty) \sim \exp(-ikz) - S \exp(ikz) \tag{14}$$

where $k = \sqrt{2mE}$ is the momentum of incident $\bar{H}$ atoms and S is the diagonal element of the $S-$matrix that describes elastic scattering. S defines the flux reflected by the asymptotic potential and the reflection coefficient $R = |S|^2$ has been obtained by numerical integration of eq. (8) with eq. (12) as a boundary condition. The resulting reflection coefficient is presented in Table I as a function of the incident energy. It is seen from the last column

that the reflection becomes considerable ($> 50\,\%$) at $E \simeq 10^{-11}$ a.u. ($T \simeq 2.7 \cdot 10^{-6}$ K) and reaches $99\,\%$ at $E \simeq 10^{-14}$ a.u. ($T \simeq 2.7 \cdot 10^{-9}$ K). The results underscore that, in spite of strong atom-wall attraction, slow (anti)atoms are reflected by and only a small fraction of the incoming flux reaches the wall.

The reflection coefficient in the low energy limit is conveniently expressed in terms of the energy independent constant $a = \lim_{k\to 0} \frac{1-S}{2ik}$, known as the scattering length. In the low energy limit the amplitude of the reflected wave S can be expanded in terms of k

$$S(k) \simeq 1 - 2ika. \tag{15}$$

Due to annihilation on the wall $|S| < 1$, the scattering length a acquires a negative imaginary part. The flux reflected in the region where the long range tail of the interaction potential dominates takes the form frequently used in the literature on quantum scattering [8]

$$|S|^2 \simeq 1 - 4k|\operatorname{Im} a| = 1 - kb \tag{16}$$

where we have introduced the constant $b = 4|\operatorname{Im} a|$. The flux, absorbed (annihilated) on the wall is

$$P(k) = 1 - |S|^2 \simeq kb. \tag{17}$$

The annihilation probability P becomes small when $kb \ll 1$, whereas the reflection probability tends to 1. We have calculated the scattering length for the potential $V_{CP}(z)$ with full absorption boundary condition,

$$a_{CP} = -52.4 - i543.5 \text{ a.u.}, \tag{18}$$

$$\operatorname{Re} a_{CP} / \operatorname{Im} a_{CP} = 0.09, \tag{19}$$

$$b = 2174.0 \text{ a.u.}. \tag{20}$$

The annihilation probability P for potential V_{CP} is expressed in terms of the scattering length and is tabulated in Table I along with the exact values. One notices from column 4 that the scattering length approximation (eq. (17) becomes valid for $E < 10^{-11}$ a.u. i.e. when $kb \ll 1$, but the validity of the non-perturbative approximation (given in column 3) extends to higher temperatures.

It is interesting to compare this value for the scattering length with the corresponding value for the purely homogeneous potential $-C_4/z^4$ with the full absorption at the origin.

$\log(E/a.u.)$	P	$1-\exp(-kb)$	kb	R
-9	0.95	0.99	4.16	0.05
-10	0.69	0.74	1.32	0.31
-11	0.33	0.34	0.42	0.67
-12	0.12	0.13	0.13	0.88
-13	0.04	0.04	0.04	0.96
-14	0.013	0.013	0.013	0.987
-15	0.0042	0.0042	0.0042	0.9958
-16	0.0013	0.0013	0.0013	0.9987
-17	0.00042	0.00042	0.00042	0.99958
-18	0.00013	0.00013	0.00013	0.99987

TABLE I: The annihilation (P) and reflection (R) probabilities for the ultra-cold antihydrogen impinging on the material wall. Column 3: non-perturbative approximation to P. Column 4: scattering length approximation to P.

In the latter case [21]

$$a_s^{abs} = \exp(-i\pi/(s-2)) \left(\frac{\sqrt{2mC_s}}{s-2} \right)^{2/(s-2)} \Gamma((s-3)/(s-2))/\Gamma((s-1)/(s-2)) \qquad (21)$$

and for $s = 4$ (the Casimir correction) the scattering length is purely imaginary and given by

$$a_4 = -i\sqrt{2mC_4} \qquad (22)$$

which for $\bar{H}$ results in

$$a_4(\bar{H}) = -i519.9 \text{ a.u.}. \qquad (23)$$

We note that the imaginary part of the scattering length a_{CP} is rather close to the value obtained for a purely homogeneous $-C_4/z^4$ potential and suggest that the main contribution to the penetration probability arises from the asymptotic Casimir tail (7). The nonzero real part of the scattering length is the contribution from distances where the potential changes from the van der Waals (6) to the Casimir limit (7). In fact, one can see from (21) that the real part of the scattering length for the purely homogeneous potential with $s = 4$ (with absorptive core) is exactly zero, while for $s < 4$ it is negative.

We investigate the influence of the inner van der Waals part of the potential in the zero energy limit. We model the exact potential $V_{CP}(z)$ by the analytically solvable potential [8]

$$V_m(z) \;=\; -\frac{C_4}{z^3(z+l)} \tag{24}$$

with $l \equiv C_4/C_3$. Such a potential has correct asymptotic behavior in the limit of big ($z \gg l$) and small ($z \ll l$) distances. The zero energy wave-function for such a potential is

$$\Phi_m(z) = \sqrt{z(z+l)}\left(H_1^{(1)}(2\sqrt{2mC_3(1/z+C_3/C_4)}) - e^{2i\delta_s}H_1^{(2)}(2\sqrt{2mC_3(1/z+C_3/C_4)})\right). \tag{25}$$

To obtain the scattering length we examine this expression in the limit $z \gg l$ to first order in terms proportional to l/z

$$\Phi_m(z) \;\sim\; z(1+l/(2z))\left(H_1^{(1)}(\xi(1+\frac{l}{2z})) - e^{2i\delta_s}H_1^{(2)}(\xi(1+\frac{l}{2z}))\right) \tag{26}$$

where $\xi = 2C_3\sqrt{2m/C_4}$. Employing a Taylor expansion and collecting terms proportional to z, the wave function becomes

$$\Phi_m(z) \sim \frac{l}{2}\left(1+\xi\frac{H_1^{'(1)}(\xi) - e^{2i\delta_s}H_1^{'(2)}(\xi)}{H_1^{(1)}(\xi) - e^{2i\delta_s}H_1^{(2)}(\xi)}\right) + z. \tag{27}$$

Now we use the fact that the asymptotic expression of the wave-function can be written in terms of the scattering length as

$$\Phi_m(z \gg l) \sim 1 - z/a_m \tag{28}$$

Comparing the last two equations we extract the scattering length from eq. (27)

$$a_m = -\frac{l}{2}\left(1+\xi\frac{H_1^{'(1)}(\xi) - e^{2i\delta_s}H_1^{'(2)}(\xi)}{H_1^{(1)}(\xi) - e^{2i\delta_s}H_1^{(2)}(\xi)}\right). \tag{29}$$

The full absorption boundary condition ($\delta_2 \gg 1$, $e^{2i\delta_s} \to 0$) leads to, in analogy with the results of (12), cancellation of all terms that include the solution $H_1^{(2)}(\xi)$

$$a_m = -\frac{l}{2}\left(1+\xi\frac{H_1^{'(1)}(\xi)}{H_1^{(1)}(\xi)}\right) \tag{30}$$

The ratio $\mathrm{Re}\, a_m / \mathrm{Im}\, a_m$ is given by

$$\mathrm{Re}\, a_m / \mathrm{Im}\, a_m = \frac{\pi}{2}\left(J_1^2(\xi) + Y_1^2(\xi) + \xi(J_1'(\xi)J_1(\xi) + Y_1'(\xi)Y_1(\xi))\right) \tag{31}$$

where $J_1(z)$ and $Y_1(z)$ are Bessel functions [19] of order 1. For $\bar{H}$ we obtain

$$a_m = -69.8 - i505.6 \text{ a.u.}, \tag{32}$$

$$\mathrm{Re}\, a_m / \mathrm{Im}\, a_m = 0.14, \tag{33}$$

$$\xi = 3.53 \tag{34}$$

The ratio $\mathrm{Re}\, a_m / \mathrm{Im}\, a_m$ contains important information on the scale parameter ξ that, in turn, determines the transition between the van der Waals and Casimir limits. In the limit of large ξ the model potential V_m becomes $-C_4/z^4$ whereas the ratio becomes $\mathrm{Re}\, a_m / \mathrm{Im}\, a_m \simeq 1/\xi \to 0$ as it should for the purely homogeneous potential $V = -C_4/z^4$. According to this treatment, we expect that for the exact potential V_{CP}, the smaller the ratio $\mathrm{Re}\, a_m / \mathrm{Im}\, a_m$, the more important the contribution from the Casimir tail to the scattering amplitude

III. PROBABILITY OF QUANTUM REFLECTION AS A FUNCTION OF DISTANCE

We now study the relative importance of different regions in which the van der Waals-Casimir potential dominates and contributes to the value of the reflected wave. We represent the wave-function in the following form [22]

$$\Phi(z) = \frac{A(z)}{p^{1/2}(z)}\left(B(z)\exp(i\int_{z_0}^{z} p(x)dx) - \exp(-i\int_{z_0}^{z} p(x)dx)\right) \tag{35}$$

where $A(z)$ and $B(z)$ are arbitrary functions that are obtained when we substitute $\Phi(z)$ into the Schrödinger equation. Here $p(z) = \sqrt{2m(E - V_{CP}(z))}$ is the local classical momentum, and z_0 is an arbitrary distance. Identification of a solution of the Schrödinger equation with the incoming or outgoing wave is unambiguous only in case when it can be represented in the semiclassical form $\exp(\pm i\int_{z_0}^{z} p(x)dx)$. If the the WKB approximation is valid in the entire range of antiatom-wall distances, the solution everywhere has the form of an incoming wave and no reflection occurs. However, for certain regions (2) of antiatom-wall separation, the WKB approximation fails (the so-called badlands). The solution of the Schrödinger equation differs from the semiclassical one and leads to the appearance of a reflected wave. Hence

the function $B(z)$ in (35) can be interpreted as a function that "converts" the semi-classical solution into the exact quantal solution and thus contains information on the reflected wave amplitude generated at *each separation distance* z.

We need to put one more condition on $A(z)$ and $B(z)$ in order to define them uniquely. Following the phase function method [22, 23] we require that

$$\Phi'(z) = iA(z)p^{1/2}(z)\left(B(z)\exp(i\int_{z_0}^{z} p(x)dx) + \exp(-i\int_{z_0}^{z} p(x)dx)\right). \tag{36}$$

As we have already mentioned, the function $B(z)$ is an amplitude for the reflected wave. The relationship between the asymptotic value $B(\infty)$ and the S-matrix can be easily established

$$S = B(\infty)\exp(2i(\int_{z_0}^{z\to\infty} p(x)dx - kz)). \tag{37}$$

Substituting eq. (35) and eq. (36) into the Schrödinger equation we obtain the first order nonlinear differential equation for $B(x)$:

$$B'(z) = \frac{p'(z)}{2p}\left(B^2(z)\exp(2i\int_{z_0}^{z} p(x)dx) - \exp(-2i\int_{z_0}^{z} p(x)dx)\right). \tag{38}$$

As long as in the limit $z \to z_s$ the wave-function should have the form given by (12) and (13) one should choose the initial condition as $B(z_s) = 0$, and in equation (38) one should put $z_0 = z_s$.

The function $|B(z)|^2$ can be interpreted as that "portion" of the reflected wave that is generated in the domain between at z_0 and z. (Such an interpretation is unambiguous when z belongs to the range where the WKB approximation is valid, and the functions $\exp(\pm i\int_{z_0}^{z} p(x)dx)$ can be identified with the reflected or transmitted waves). Obviously $|B(\infty)|^2$ gives the "full" reflection probability for the given energy. In Fig.1 we plot the function $|B(z)|^2$ for two energies $E = 10^{-12}$ a.u. and $E = 10^{-10}$ a.u.

As $z \to 0$ the WKB approximation becomes valid. Using it (and thus reflection from these parts of potential exponentially decreases) we found that the contribution to the amplitude of the reflected wave from the distances $z < 100$ a.u. is indeed small.

In addition, at ultra-low energies (practically for $E \ll 10^{-10}$ a.u.) the reflected wave is generated within a wide range of distances. In that case we cannot define an unambiguous "reflection distance". The plot shows that for the energy $E = 10^{-12}$ a.u. the reflection probability is 0.88 and this value is in harmony with the calculated value presented in Table I. We note that 75% of the reflected wave amplitude is generated in the domain from 500 a.u.

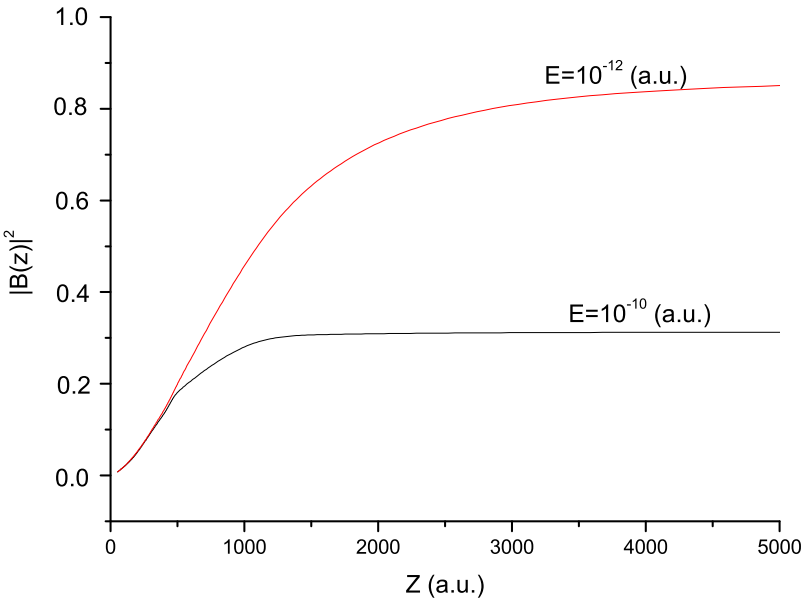

FIG. 1: Contribution of different antiatom-wall distances to the reflection probability. $|B(z)|^2$ expresses the reflection probability accumulated in the interval between z_0 and z.

to 5000 a.u., the region where the Casimir tail of the potential dominates. At intermediate distances, from 100 a.u. up to 500 a.u., the contribution is 22%. However, with increasing energy the reflecting domain is more localized and is shifted to shorter distances, whereas the reflection probability is diminished. According to Fig.1, for $E = 10^{-10}$ a.u. the reflection probability is 0.31 and 92% of the reflected wave is generated in the domain from 100 a.u. up to 1000 a.u. In this case the reflection probability reaches its full value within a well defined interval Δz around z_r, and allows us to define z_r, the "reflection distance". At higher energies, reflection occurs at the reflection distance ($\Delta z \ll z_r$) and the reflection probability becomes exponentially small. The reflection coefficient as well as reflection distance z_r can be estimated using "complex turning point method" [8, 24, 25].

As discussed above, the real part of the scattering length is more sensitive to the details of the potential at intermediate distances. More generally, one can expect that the real part of the phase shift $\delta(E)$ (defined through the equality $S = \exp(2i\delta(E))$) provides additional information on the antiatom-surface interaction. In Fig. 2 we plot $\mathrm{Re}\,\delta(E)$ for the exact interaction $V_{CP}(z)$, as well as for two model interactions $V_1 = -C_4/z^4$ and $V_2 = -C_4/(z^4 +$

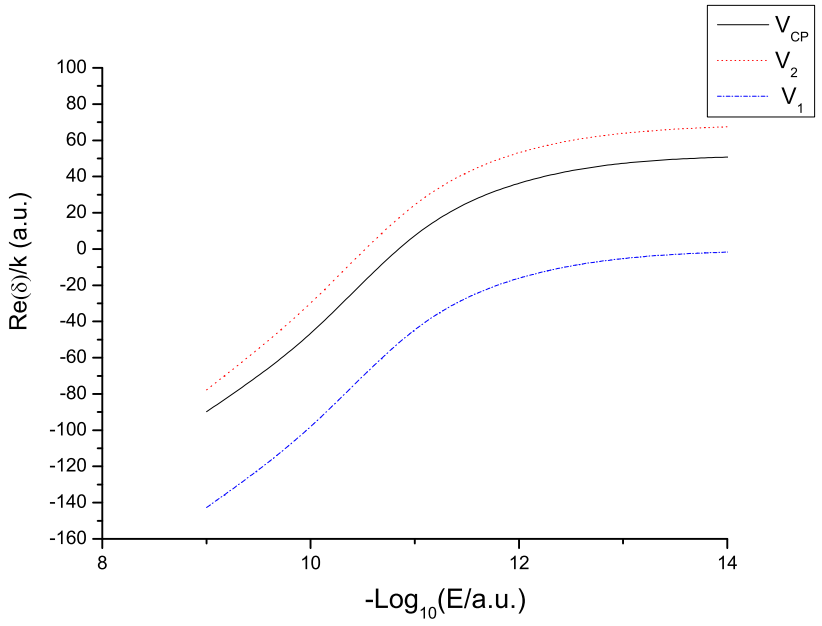

FIG. 2: $\mathrm{Re}\,\delta/k$ as a function of energy for exact interaction V_{CP} (solid line) and for model interactions V_1 (dashed line) and V_2 (dotted line).

z^3C_4/C_3).

We found that the exact phase shift is negative for $E > 10^{-9}$ a.u. and positive for $E < 10^{-9}$ a.u. The phase shift produced by the homogeneous potential $V_1 = -C_4/z^4$ is always negative and tends to 0 in the limit $E \to 0$. Note that in the limit of zero energy $\mathrm{Re}\,\delta(E \to 0)/k = -\,\mathrm{Re}\,a_{CP}$.

In conclusion, we emphasize that annihilation of $\bar{H}$ on the surface allows a description that depends only on the nature of the van- der Waals-Casimir potential. In particular all inelastic processes that occur at characteristic distances of few a.u. (including thermal heating of $\bar{H}$ atoms by phonon exchange [26, 27]) do not affect the reflected wave.

In the case of strong absorption the phase and amplitude of the reflected wave are independent inn details of the short-range interaction. This allows one to glean important information on parameters that characterize the van der Waals-Casimir atom-wall potential. In the limit of a perfectly conducting surface, they are solely determined by the dynamical dipole polarizability of $\bar{H}$. Our treatment can be generalized to the case of finite conductivity or non-zero temperatures by introducing corrections to the Casimir-Polder interaction

constants [18]. In particular, the simple estimation of $\bar{H}$ quantum reflection constant b in case of dielectric surface can be obtained using expression (22) with the modified value of C_4 coefficient:

$$b \approx 4\sqrt{2mC_4\frac{\epsilon-1}{\epsilon+1}F(\epsilon)}$$

where ϵ is permittivity of the dielectric material. The correction function $F(\epsilon)$ calculated in [6] is approximately equal to 0.77 in a wide range of ϵ. The estimated in such a way constant b for polyethylene ($\epsilon \approx 2.3$) is $b \approx 1138.9$ a.u., which is approximately twice smaller than in case of perfect conducting surface. As a consequence the characteristic energy, for which quantum reflection from polyethylene surface starts to be significant ($kb \sim 1$, $E = 1/(2mb^2)$) is four times greater, than in case of perfect conducting surface.

In the following section we consider the confinement of antihydrogen between material walls. In particular we show how the properties of quantum reflection are related to the shifts and widths of the energy levels of $\bar{H}$ atoms enclosed in cavities with conducting walls.

IV. ULTRACOLD $\bar{H}$ BETWEEN TWO CONDUCTING WALLS

The partial reflection of ultra-cold $\bar{H}$ from the material wall enables the existence of quasi-stationary (decaying) states of $\bar{H}$ in-between two walls. In so far we will be interested in the one-dimensional wave-function of such states. We expect that the distance L between the conducting walls is large compared to $|a_{CP}|$ so that the atom interacts with each of the walls independently. The boundary condition at the left wall ($z = 0$) is given by (12). To obtain the boundary condition at the right wall one should replace in equation ([**?**]) z by $L - z$. Far from the walls ($|a_{CP}| \ll z \ll L - |a_{CP}|$) the wave function is

$$\Phi(z) \sim \sin(kz + \delta_{CP}) \tag{39}$$

where δ_{CP} is the phase-shift produced by interaction with the wall. Using the symmetry of the problem one obtains the quantization condition

$$kL + 2\delta_{CP} = \pi n. \tag{40}$$

We consider the low energy domain where that the scattering length approximation is valid, so that $\delta_{CP} = -ka_{CP}$. Thus we get for k

$$k = \pi n/(L - 2a_{CP}), \tag{41}$$

and so the energies of the box-like states are quantized. The eigenvalues are modified by quantum reflection according to

$$\mathcal{E}_n \equiv \varepsilon_n - i\frac{\Gamma_n}{2} = \frac{\pi^2 n^2}{2m(L-2a_{CP})^2} \simeq \frac{\pi^2 n^2}{2mL^2}(1 + 4\frac{\mathrm{Re}\, a_{CP}}{L}) - i|\,\mathrm{Im}\, a_{CP}|\frac{4\pi^2 n^2}{2mL^3}. \tag{42}$$

The widths of the states are given by

$$\Gamma_n = 2b\frac{\pi^2 n^2}{2mL^3} = \frac{2\varepsilon_n^{(0)} b}{L} \tag{43}$$

where $\epsilon_n^{(0)} = \pi^2 n^2/(2mL^2)$ are the energy levels unperturbed by the long range $\bar{H}$ – wall interaction.

The level shifts and widths are determined by the value of *complex* scattering length ($b = 4|Ima|$, c.f. eq. 18) that characterizes quantum reflection. The lifetimes increase with the wall separation as L^3 and decrease with the excitation quantum number as n^2. These expressions are obtained using the assumption that the scattering length approximation (15) is valid, and restricts its validity for $\varepsilon_n < 10^{-11}$ a.u.

The solutions $\Phi_n(z)$, corresponding to the complex energy levels $\mathcal{E}_n$, are the eigenfunctions of a non-self-adjoint Hamiltonian. In fact, they are decaying quasi-bound states with energy $\varepsilon_n - i\Gamma_n/2$. This implies that such states obey the bi-orthogonality condition

$$\int_0^\infty \Phi_n(z)\Phi_k(z)dz = \delta_{nk} \tag{44}$$

and differs from the "standard" expression by the absence of complex conjugation of the $\langle$bra| function.

The expression for the width (43) coincides with the simple formula for the annihilation rate of the particles moving freely with the velocity $v = \sqrt{2\varepsilon^{(0)}/M}$ between the walls separated by the distance L

$$\Gamma = P(v)\omega, \tag{45}$$

where $P(v)$ is the wall penetration probability and $\omega = v/L$ is the frequency of "hits" on the wall. Substituting the probability of annihilation on the wall $P(v)$ from (17) we again

obtain the expression given in (43), i.e. $\Gamma = 2\varepsilon^{(0)}\frac{b}{L}$. At high energies ($\varepsilon > 10^{-8}$ a.u.) the annihilation probability $P(v) \simeq 1$, and we get $\Gamma = \frac{v}{L}$. The lifetime of such a "fast" particle is just its time of flight between the walls.

As an example, we take $L = 10\ \mu m$ which is much greater than the length scale $b = 0.115$ μm, that characterize the Casimir force. At this wall separation distance, the ground state energy is $\varepsilon_0 = 7.5\ 10^{-14}$ a.u. the width is $\Gamma_0 = 1.7\ 10^{-15}$ a.u. and corresponds to the lifetime 0.014 s. We compare this value to the time of flight between the walls $t = L/v = 0.0005$ s. Thus $\bar{H}$ in this state bounces about 30 times before it annihilates. Since the life time is proportional to L^3, and the passage time to L^2, the number of the bounces between the walls grows linearly with L. Let us mention here that the concept of quasi-stationary state itself is meaningful only in case its life-time is much greater than the corresponding time of flight between the walls.

V. ANTIHYDROGEN IN A WAVE-GUIDE

A laboratory demonstration of quantum reflection states could realized by introducing $\bar{H}$ atoms through a slit between parallel conducting walls separated by distance L. (The similar principle was used to observe quantum motion of neutrons in the gravitational field of Earth [28, 29]. Ignoring, for the moment, the influence of gravitation on $\bar{H}$, we align the conducting walls parallel to the gravitational field. The Schrödinger equation, that governs the motion of antiatoms inside the wave-guide with transverse dimension z and horizontal dimension x, is

$$\left[-\frac{\partial^2}{2m\partial x^2} - \frac{\partial^2}{2m\partial z^2} + V(z) + V(L-z) - E\right]\Psi(z,x) = 0. \tag{46}$$

We express the two-dimensional wave-function $\Psi(z,x)$ as a series of products of normalized transverse wave-functions $\Phi_n(z)$ given in eq. (39) and longitudinal plane-wave functions $\exp(ip_n x)$:

$$\Psi(z,x) = \sum_n C_n \exp(ip_n x)\Phi_n(z) \tag{47}$$

where C_n are the amplitudes of corresponding states, dependent on the properties of the flux entering the wave-guide, and p_n is the horizontal momentum of the state with the

transverse energy $\varepsilon_n - i\Gamma_n/2$, so that

$$p_n^2/2m + \varepsilon_n - i\Gamma_n/2 = E. \tag{48}$$

Let us mention that the eigen-functions $\Phi_n(z)$ of not-self-adjoint Hamiltonian do not form a basis in the Hilbert space [30]. So far the expansion (47) is an approximation. We will show however that under certain conditions of the wave-guide experiment few quasi-stationary states from (47) would be give exhaustive contribution to the measured flux, which verifies our approach.

If the characteristic spatial dimension of the incoming flux H_0 is much larger than the separation L and the distribution of the transverse velocity in the incoming flux exceeds the value $1/L$, the first few excited states are uniformly distributed. Because the transverse energy $\varepsilon_n - i\Gamma_n/2$ is complex, the horizontal momentum p_n, for a given state, is also complex (c.f. 48).

The horizontal momentum at small transversal energy ($\varepsilon_n \ll E$) is given by

$$p_n \simeq \sqrt{2mE} - \sqrt{m/2E}(\varepsilon_n + i\Gamma_n/2) = p - \frac{(\varepsilon_n - i\Gamma_n/2)}{v} \tag{49}$$

where $p = \sqrt{2mE}$ and $v = p/m$. We can thus write for the wave-function $\Psi(z,x)$

$$\Psi(z,x) = \exp(ipx) \sum_n C_n \exp(-\Gamma_n x/(2v)) \exp -i\epsilon_n x/v \Phi_n(z). \tag{50}$$

Using the definition $t = x/v$ we relate the decay time of the transverse states $\Phi_n(z)\exp(-\Gamma_n t/2)$ to the disappearance rate (along the x-axis) of the flux density inside the wave-guide.

The integrated flux density $F = F_0 \int |\Psi(z,d)|^2 dz$ (where F_0 is a normalization constant) gives the number of counts of $\bar{H}$ in the detector at the exit ($x = d$) of the wave-guide

$$\begin{aligned} F &= F_0 \sum_n |C_n|^2 \langle \Phi_n | \Phi_n \rangle \exp(-\Gamma_n \tau^{pass}) \\ &+ F_0 \sum_{n \neq k} C_n^* C_k \langle \Phi_n | \Phi_k \rangle \exp(i(\varepsilon_k - \varepsilon_n)\tau^{pass}) \exp(-\frac{(\Gamma_n + \Gamma_k)\tau^{pass}}{2}) \end{aligned} \tag{51}$$

where we have introduced the passage time $\tau^{pass} = d/v$.

We denote the second term in this expression as an "interference" term since its presence is due to non-orthogonality of the decaying states, $\langle\Psi_n|\Psi_k\rangle \neq \delta_{nk}$. The appearance of such a term is not surprising. Indeed, decaying states are not stationary and they don't have well defined energies and so they can be thought of as a superposition of truly stationary states. Therefore transitions between states with frequencies $\omega_{nk} = \varepsilon_k - \varepsilon_n$ occur. The transitions manifest in the appearance of the oscillating exponents $\exp(i\omega_{nk}\tau^{pass})$ in the expression for the measured flux (51). However, observation of such interference requires rather good resolution in the horizontal velocities of the initial flux. The uncertainty in passage time $\delta\tau^{pass} = \tau^{pass}\delta v/v$ is of order of ω_{nk}^{-1}, and it follows that the limit for horizontal velocity resolution is

$$\frac{\delta v}{v} < \tau^{pass}\omega_{nk}. \tag{52}$$

For a broader distribution in the horizontal velocity, the interference terms cancel due to fast oscillating exponents and the expression for the flux takes the form

$$F = F_0 \sum_n \exp(-\Gamma_n \tau^{pass}). \tag{53}$$

The time of flight τ^{pass} may be chosen in such a way that only a few transverse states with the smallest $\Gamma_n = n^2\Gamma_1$ contribute to the flux of $\bar{H}$ in the wave-guide. This time of flight selects the states with small transversal energy ε_n and thus our expression for the horizontal momentum (49) is justified. Let us also note that such a choice of time of flight ensures that contribution of quasi-stationary states to the flux at the exit of the wave-guide is much greater than the contribution of all the "non-resonant" terms, which were neglected in the approximative expansion (47) of the total wave-function [31]. The typical length d for a flux of $\bar{H}$ with the horizontal velocity $v = 10$ m/s should be about 10 cm to ensure that only the ground transverse state passes through the slit $L = 10\,\mu m$. Changing the slit separation leads to an enhanced contribution from the excited transversal states, due to the dependence (43) of the decaying rates Γ_n on L. The measured function $F(L)$ provides information on the parameter b. In particular, if only the ground state passes through the guide, one gets the following (C_1 and F_0 independent) expression for b

$$b = \left(\frac{\partial \ln F(L)}{\partial L}\right) \frac{mL^4}{3\pi^2\tau^{pass}}. \tag{54}$$

We stress that the quantity b above is very sensitive to the Casimir force since quantum reflection in the guide selects only those antiatoms that have interacted with the surface through the van der Waals - Casimir potential (all others are annihilated).

VI. GRAVITATIONAL EFFECTS

We have shown that quantum reflection takes place for very slow anti-atoms. This fact could be exploited to study the gravitational properties of anti-atoms. In this section we discuss the possibility for measuring the gravitational force acting on antihydrogen. We distinguish between the gravitational mass, which we refer to as M and inertial mass, hereafter denoted by m. We consider an $\bar{H}$ atom bouncing above a conducting surface in the gravitational field of Earth. Confinement is achieved by quantum scattering by the van der Waals-Casimir potential from below, and by the gravitational field from above. The characteristic length and energy scales are

$$l_0 = \sqrt[3]{\frac{\hbar^2}{2mMg}}, \tag{55}$$

$$\varepsilon = \sqrt[3]{\frac{\hbar^2 M^2 g^2}{2m}}. \tag{56}$$

The characteristic scale of the gravitational energy of $\bar{H}$ is $\varepsilon = 2.211\ 10^{-14}$ a.u., whereas the corresponding length scale is $l_0 = 5.871\ \mu m$ and is much greater than the length scale, ($b = 0.115\ \mu m$), of the Casimir force. Thus, the gravitational field does not significantly affect the atom-wall interaction. The energy levels and the corresponding wave-functions are

$$E_n = \varepsilon \lambda_n, \tag{57}$$

$$\Phi_n(z) = C\,\mathrm{Ai}(z/l_0 - \lambda_n) \tag{58}$$

where $\mathrm{Ai}(x)$ is the Airy function [32] and the eigenvalues λ_n are found by enforcing the condition $\mathrm{Ai}(-\lambda_n) = 0$. The modification in the values for the eigenvalues λ_n due to quantum reflection are obtained by matching the wave-function of $\bar{H}$ reflected from the wall (which at the distance $|a_{CP}| \ll z \ll l_0$ has the asymptotic form $\Phi(z) \sim 1 - z/a_{CP}$) to the gravitational wave-function $\mathrm{Ai}(z/l_0 - \widetilde{\lambda})$, where $\widetilde{\lambda}$ is a modified eigenvalue. Taking into account that in the matching region $z/l_0 \ll 1$, we obtain the following equation for $\widetilde{\lambda}$

$$\frac{\mathrm{Ai}(-\widetilde{\lambda})}{\mathrm{Ai}'(-\widetilde{\lambda})} = -a_{CP}/l_0. \tag{59}$$

Because $a_{CP}/l_0 \ll 1$, $\widetilde{\lambda}$ can be expressed as a perturbation series in terms of the gravitational eigenvalues λ_n

$$\widetilde{\lambda}_n = \lambda_n + \delta\lambda_n \tag{60}$$

with $\delta\lambda_n \ll 1$. This leads an expression for the modified eigenvalues $\widetilde{\lambda}_n$

$$\widetilde{\lambda}_n = \lambda_n + a_{CP}/l_0. \tag{61}$$

From it, we obtain modified energy levels and the corresponding widths

$$E_n \;=\; \varepsilon(\lambda_n + \frac{\mathrm{Re}\, a_{CP}}{l_0}), \tag{62}$$

$$\Gamma_n \;=\; \varepsilon\frac{b}{2l_0} \tag{63}$$

The widths of the gravitational states (63) are proportional to the combination ε/l_0. Using (55) and (56) we find that this ratio is equal to the gravitational force $\varepsilon/l_0 = Mg$ so that

$$\Gamma_n = \frac{b}{2}Mg. \tag{64}$$

The value of the ground state energy of $\bar{H}$ in the gravitational field of Earth turns out to be $E_1 = 5.17\ 10^{-14}$ a.u. The corresponding life time is

$$\tau = \frac{2\hbar}{Mgb} \simeq 0.1\mathrm{s}. \tag{65}$$

We note factorization of gravitational effects (appearing in the above formula via factor Mg) with the effect of quantum reflection, appearing through the constant b. Such a factorization is a consequence of the small ratio in the characteristic scale $b/l_0 \simeq 0.02$.

We find that the widths of the gravitational states (63) are independent of the energy (for the energies $E_n < 10^{-11}$ a.u.) to the first order in the small ratio b/λ_n. This is understood using the following simple argument. The frequency of the atom bouncing above the surface in the gravitational field is $\omega \sim 1/\sqrt{E}$, whereas the probability (17) of annihilation on the

wall is $P \sim \sqrt{E}$. Combining the two, we obtain an energy independent expression for the width, $\Gamma = \omega P$. That is, all $\bar{H}$ atoms with $E_n < 10^{-11}$ a.u. bouncing on the surface survive for 0.1 s.

The measurement of this lifetime can be achieved by monitoring the spatial decay of the flux of antiatoms with the velocity v parallel to the surface. Such a flux, as function of the distance d and integrated over vertical dimension z is

$$F(d) = F_0 \exp(-\frac{\varepsilon}{\hbar}\frac{b}{2l_0}\tau^{pass}) = \exp(-\frac{Mgb}{2\hbar}\tau^{pass}). \tag{66}$$

If one shifts the position d of the detector and measures the spatial decay $F(d)$, one can infer the gravitational force Mg acting on the antiatom since

$$Mg = -2\frac{\partial \ln F(d)}{\partial d}\frac{\hbar v}{b}. \tag{67}$$

VII. SURFACE STATES FOR ATOMS AND ANTIATOMS

Our treatment of quantum reflection of antiatoms makes use of the fact that the antiatom-wall interaction is strongly absorptive at short distance, and is mathematically expressed by absorption condition (11). In this section we study the case where absorption on the surface is not necessarily strong. Such a situation may correspond to the partial loss of ordinary atoms that undergo inelastic interaction at the surface, or to reflection of antiatoms from partially transparent evanescent-wave atomic mirror.

Below we outline a resonance-type dependence of the atom loss probability on the properties of the short range interaction in case of the weak surface absorption. We show that such resonance behavior stems from the presence of narrow near-threshold atom-surface states [33].

For the purpose of illustration, and in order to allow an analytic description, we replace the exact van der Waals-Casimir potential by the model potential (24). We do not impose the full absorption boundary condition, instead the *complex* phase-shift δ_s, produced by the short-range interaction is used.

For the model potential V_m, the analytic expression for the scattering length (29) simplifies substantially since $\xi = 2C_3\sqrt{2m/C_4} \gg 1$. and using the asymptotic representation of the

Hankel functions $H_1^{(1,2)}(\xi) \sim \sqrt{2/(\pi\xi)}\exp(\pm i(\xi - 3\pi/4)$ we obtain

$$a_m = -\frac{l}{2}\xi\cot(\xi - \delta_1 - i\delta_2 - 3\pi/4). \tag{68}$$

We explore the dependence of a_m on the real part of the short-range phase-shift δ_1. We find that a_m is an oscillating function of δ_1. The smaller the imaginary part δ_2, the more prominent are the oscillations. The maxima occur when

$$\delta_1^{res} = \xi - 3\pi/4 - \pi n,\ n = 1, 2, ... \tag{69}$$

This expression is simply the condition for the appearance of a new state in the potential well of an attractive van der Waals– Casimir potential and a short-range core.

The existence of the narrow near-threshold state leads to a probability loss $P = 4k|\operatorname{Im} a_m|$ that varies rapidly with variations in the phase δ_1. At its maximum, the probability loss (for the above mentioned case when $\xi \gg 1$ and $\delta_2 \ll 1$) is

$$P_{max} = 2kl\frac{\xi}{\delta_2} \tag{70}$$

and the width of such a maximum is equal to δ_2.

This example illustrates the influence of near-threshold singularities on the S-matrix and scattering observables. To find the position of the S-matrix poles in the complex k-plane, we express the S-matrix in the vicinity of its pole

$$S = \frac{z + k}{z - k}. \tag{71}$$

Taking into account that for small $k \ll |z|$ we have $S = 1 - 2ika$, we get for the *near-threshold* pole position

$$z = i/a. \tag{72}$$

Using a Taylor expansion of the scattering length a_m (68) in the vicinity of the resonance δ_1^{res}, the position of the near-threshold pole can be written as

$$z \simeq \frac{2i}{l\xi}\Delta\delta_1 - \frac{2\delta_2}{l\xi} \tag{73}$$

where $\Delta\delta_1 = \delta_1 - \delta_1^{res}$ is the deviation of the short-range phase-shift from the resonance value.

The maximum in the loss probability occurs when the pole z of the S-matrix crosses the real axis of the complex k-plane. In the absence of absorption, the pole lies on the imaginary axis of the complex k-plane and when it crosses the real axis the scattering length approaches positive infinity. In the presence of absorption the pole has a real negative component. The smaller the shift from the imaginary axis, the more pronounced the effect.

As an illustration, we consider the values δ_1 and δ_2 obtained within the model potential

$$V_s(z) = \begin{cases} U\exp(-i\varphi)\exp\left((z_0-z)/\rho\right) & \text{if } z \geq z_0 \\ U\exp(-i\varphi) & \text{if } z < z_0 \end{cases} \tag{74}$$

Here the argument of the complex exponent φ relates the real and imaginary part of the complex potential and ρ is the diffuseness radius. This model allows analytic solutions and facilitates our understanding of inelastic processes nearby and inside the wall.

For a strong complex exponential potential V_s where $|\operatorname{Im}\rho\sqrt{2MU}| \gg 1$, the scattering length is [34]

$$a_s \simeq z_0 + \rho\left(2\gamma + 2\ln(\rho\sqrt{2m|U|}) - i\varphi\right) \tag{75}$$

where $\gamma \simeq 0.577$ is the Euler constant.

We first consider the case where the diffuse radius ρ of the exponential potential is much smaller than the local de Broglie wavelength $\lambda_B(z_0) = 1/\sqrt{2mC_3/z_0^3}$ at the distance z_0. Here the short range potential V_s varies rapidly when compared to variations of the de Broglie wavelength. At distances $\rho \ll (z-z_0) \ll \lambda_B(z_0)$ the wave-function is

$$\Psi_s = 1 - \frac{z-z_0}{a_s - z_0}. \tag{76}$$

Matching it with wave-function (25) and taking into account the asymptotic properties of the Hankel function with a large argument

$$\xi_0 \equiv 2\sqrt{2mC_3(1/z_0 + C_3/C_4)}) \gg 1 \tag{77}$$

we obtain

$$\delta_1 \simeq \xi_0 - 3\pi/4 \tag{78}$$

$$\delta_2 = \varphi\rho/\lambda_B(z_0) = \varphi\rho\sqrt{2MC_3/z_0^3}. \tag{79}$$

where we have kept only the leading terms in $\rho/\lambda_B(z_0)$.

Thus, the limit $\rho/\lambda_B(z_0) \to 0$ implies vanishing absorption ($\delta_2 \to 0$) and corresponds to the case where the reflecting wall is placed at position z_0.

The weak absorption limit $\delta_2 \ll 1$ can be realized by an optical evanescent wave that generates an effective repulsive potential. The detailed study of this case is beyond the scope of our paper; for the purpose of our qualitative analysis it is only important that the imaginary part of the phase-shift δ_2 turns to be small when $\rho/\lambda_B(z_0) \ll 1$. At the same time the phase-shift δ_1 is well approximated by expression (78).

In the case $\rho/\lambda_B(z_0) \geq 1$, the short range potential varies slightly with respect to the scale of change of λ_B and one can use the semiclassical expression for the phase-shift

$$\delta_2 = \mathrm{Im} \int_{-\infty}^{\infty} \sqrt{-2m(V_s(z) + V_m(z))} dz \tag{80}$$

Thus a shallow but extended imaginary potential can produce a large imaginary phase-shift $\delta_2 \gg 1$. A a necessary condition for the validity of the strong absorption boundary condition is that the short-range interaction varies slowly in comparison with the variation in the de Broglie wavelength $\lambda_B(z_s)$ at z_s, the distance where the short-range interaction is important. For $\bar{H}$ the characteristic de Broglie wavelength at $z_s \simeq 1$ a.u. is $\lambda_B(z_s) \sim 0.05$ a.u. and the scale ρ of the short-range inelastic interactions is about 1 a.u. Well within the range where the strong absorption boundary condition is valid.

We substitute (78) into (68) to get

$$a_m = -\frac{l}{2}\xi \cot(\xi - \xi_0 - i\delta_2) \tag{81}$$

where $n = (\xi_0 - \xi)/\pi$ is the number of atom-surface bound states.

The scattering length a_m exhibits resonance behavior when the position of the "reflecting wall" is chosen in such a way that a new state appears at threshold, i.e. $\xi_0(z_0) - \xi \to \pi n_0$. In the vicinity of z_0 it is useful to represent the scattering length (81) in terms of a Breit-Wigner form

$$\mathrm{Im}\, a_m = -\frac{l\xi}{2} \frac{\delta_2}{(\xi - \xi_0 - \pi n_0)^2 + \delta_2^2}, \tag{82}$$

$$\mathrm{Re}\, a_m = -\frac{l\xi}{2} \frac{\xi - \xi_0 - \pi n_0}{(\xi - \xi_0 - \pi n_0)^2 + \delta_2^2}. \tag{83}$$

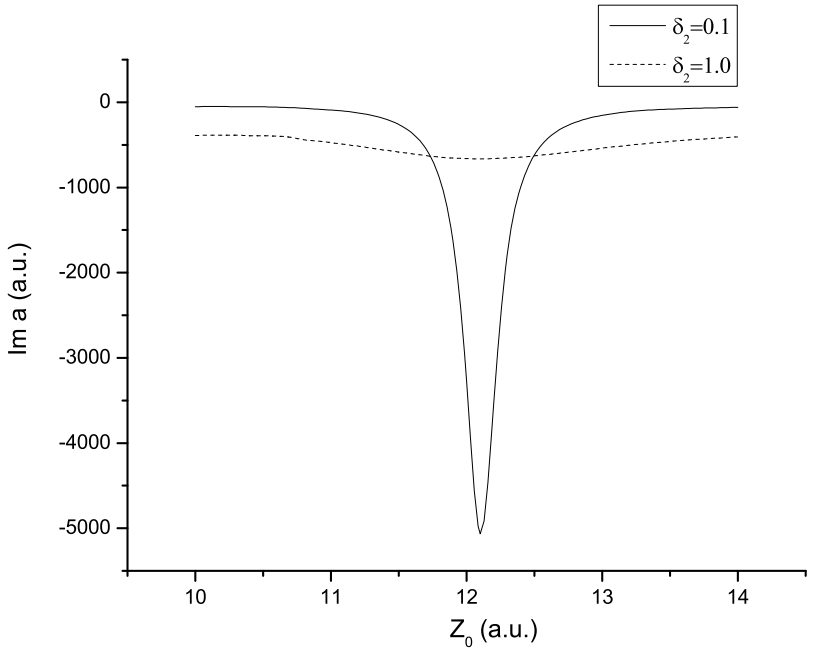

FIG. 3: Imaginary part of the scattering length as a function of the position of the "reflecting wall".

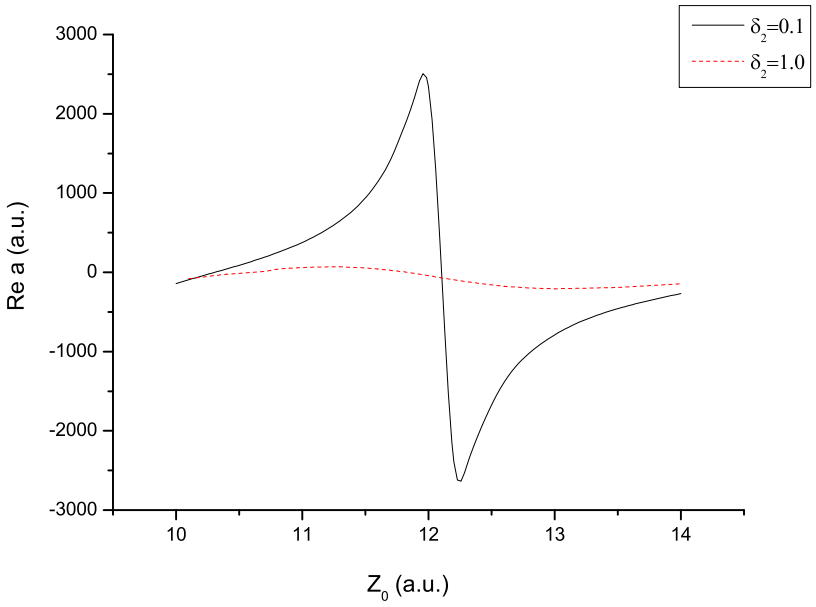

FIG. 4: Real part of the scattering length as a function of the position of the "reflecting wall".

In Fig. 3 we plot the imaginary part of a_m as a function of z_0 for two values of the imaginary phase-shift $\delta_2 = 0.1$ and $\delta_2 = 1.0$. We vary z_0 from 10 a.u. to 15 a.u. Fig. 4 illustrates the behavior of the real part of the scattering length for the two values of δ_2 given above. We find a resonance-like increase in atom loss when the short range potential is centered around $z_0 = 12.1$ a.u. This resonance is very prominent for weak absorption when $\delta_2 = 0.1$, and it is practically washed out in case of stronger absorption $\delta_2 = 1.0$.

In the hypothetical case where the short range part of the (anti)atom-wall interaction can be varied in a controlled way (e.g. by reflection from the optical evanescent wave) one could accurately measure the properties of the (anti)atom-wall potential by careful tuning of resonances and fitting the experimental data to eq. (82). In this way, one could measure the values of the C_3 and C_4 coefficients.

In conclusion, we find that the main difference in quantum reflection in cases of strong and weak absorption by the material wall is that in the latter case, narrow atom-surface states can be formed. The position of these states depends on the short range interaction. The presence of a near-threshold state strongly enhances the probability of atom loss, and might be used for resonant spectroscopy (e.g. with the use of optical evanescent wave). In case of strong absorption, the near-threshold states are completely destroyed by annihilation.

VIII. CONCLUSIONS

We presented a theoretical account of cold antiatoms interacting with a material wall. We focused on the antihydrogen - wall system where the latter is represented by an ideally conducting surface. We have shown that, as antiatoms approach the surface in an attractive potential they are not necessarily annihilated. If sufficiently cold, they may be totally reflected. Reflection occurs in the region where the long range tail of the van der Waals - Casimir interaction dominates. We calculated the reflection probability as function of the incident energy of antiatoms. We analyzed the behavior of the reflected wave as a function of distance from the surface and introduced the notion of a "reflection distance". We have also shown that quantum reflection allows the existence of long-lived metastable states of $\bar{H}$ in the conducting cavity.

We pointed out that scattering of ultra-cold $\bar{H}$ in an attractive van der Waals - Casimir potential is sensitive to retardation in the antiatom-wall interaction. We explored the possi-

bility of performing experimental measurements with $\bar{H}$ by introducing it into a wave-guide. The rate of the $\bar{H}$ annihilation on the wall of the guide could provide important information on retardation effects in matter-antimatter systems. In the strong absorption limit, such measurements are not "contaminated" by effects produced on the wall and which are characterized by poorly known short-range parameters. However, they could be employed to study the asymptotic properties of $\bar{H}$- wall interaction.

We also proposed measurement of effects due to gravity acting on the antihydrogen. In particular we found that the life-time of $\bar{H}$, in the combined gravitational and the mirror image potential of a conducting plate, to be on the order of 0.1 s. The established dependence of that lifetime, $\tau = 2\hbar/(Mgb)$, on the gravitational force allows the possibility for measuring the gravitational properties of antimatter.

Finally, we outlined the role of the near-threshold atom-surface states the where absorption on the surface is small. For antiatoms, weak absorption could arise with the introduction of an evanescent wave that prevents antiatoms from direct contact with the wall. We showed that a resonant increase in the absorption rate can be observed if the atom-wall interaction is "tuned" to form an atom-wall state at the threshold. We note the possibility of exploiting resonant absorption for studies of atom-surface interactions.

IX. ACKNOWLEDGMENT

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